Truth-Tables

Supplement
to

The Pocket Guide to Critical Thinking

and

Critical Thinking

Richard L. Epstein

Distributed *FREE*

<www.ARFbooks.org>

February 2017

ARF

Advanced Reasoning Forum

Truth-Tables

A. Symbols and Truth-Tables

The ancient Greek philosophers were the first to analyze arguments using compound claims. From then until the mid-19th century the analysis of compound claims was not much different from what you saw in Chapter 9, though many more valid and invalid argument forms were catalogued with Latin names attached. In the early 1900s, a simple method was devised for checking whether an argument form using compound claims is valid, which we'll see shortly.

We can analyze many arguments using compound claims by concentrating on how compound claims can be built up from just four English words and phrases:

and
or
not
if . . . then . . .

These words are used in many different ways in English, too many for us to investigate every possible way they could be used in arguments. We'll concentrate on *how compound claims that use them depend on the truth or falsity* (**truth-value**) *of the claims from which they are built*. We won't care how plausible a claim is, how we might happen to know it, its subject matter, or any other aspect of it.

> ***The Classical Abstraction*** The only aspects of a claim we'll pay attention to are whether the claim is true or false and how it is compounded from other claims.

To remind us that we're making this assumption, we're going to use special symbols to represent the words we're interested in.

$\wedge$	and
$\vee$	or
$\neg$	not
$\rightarrow$	if . . . then . . .

So we will replace "Spot is a dog and Puff is a cat" with "Spot is a dog $\wedge$ Puff is a cat", using our abstraction of "and." We replace "Spot is not a cat" with "$\neg$ (Spot is a cat)". Similarly, we'll use "Spot is a dog $\vee$ Puff is a cat" in place of "Spot is a dog or Puff is a cat." And for "If Spot is a dog then Puff is a cat" we'll use "Spot is a dog $\rightarrow$ Puff is a cat." If A and B are claims, we call:

A $\wedge$ B a ***conjunction***

A $\vee$ B a ***disjunction***

$\neg$ A a ***negation***

A $\rightarrow$ B a ***conditional***

We need to be clear about how we will understand these words in reasoning, relative to the classical abstraction.

To start, when is "Spot is a dog and Puff is a cat" true? When both "Spot is a dog" is true and "Puff is a cat" is true. That's the only way it can be true. Let's summarize that for all conjunctions in a table:

A	B	A $\wedge$ B
T	T	T
T	F	F
F	T	F
F	F	F

Now consider "Spot is not a dog." That's true just in case "Spot is a dog" is false. Generally, we have:

A	$\neg$A
T	F
F	T

For disjunctions, consider "London is the capital of England or Paris is the capital of France." Is this true? Some say it isn't, because "London is the capital of England" and "Paris is the capital of France" are both true. Others say the compound is true. The question is whether an "or" claim can be true if both parts are true. It turns out to be simplest to use $\vee$ to formalize "or" in the *inclusive* sense: one or the other or both parts are true. Later we'll see how to formalize "or" in the *exclusive* sense: one or the other but not both parts are true.

A	B	A $\vee$ B
T	T	T
T	F	T
F	T	T
F	F	F

The words "if . . . then . . ." have so many uses in English that it's hard to remember that we're going to pay attention only to whether the parts of the compound claim are true or false. We'll evaluate conditionals with the following table.

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

Why this table? Let's look at it row by row.

Recall that the Direct Way of Reasoning with conditionals (also called *modus ponens*) is valid:

If A then B; A; therefore B .

So if A → B is true, and A is true, then B is true (*the first row*).

Suppose A is true and B is false (*the second row*). In the valid form of *modus ponens* we can't get a false conclusion (B) from true premises. Since there are only two premises, it must be that A → B is false.

But why should A → B be true in the last two rows? Suppose Dr. E says to Suzy,

If you get 90% on the final exam, you'll pass this course.

It's the end of the term. Suzy gets 58% on the final. Dr. E fails her. Can we say that Dr. E lied? No. So the claim is still true, even though the antecedent is false and the consequent is false (*the fourth row*).

But suppose Dr. E relents and passes Suzy anyway. Can we say he lied? No, for he said "if", not "only if." So the claim is still true, even though the antecedent is false and the consequent is true (*the third row*).

The formalization of "if . . . then . . ." in this table is the best we can do when we adopt the classical abstraction. We deal with cases where the antecedent "does not apply" by treating the claim as vacuously true.

B. The Truth-Value of a Compound Claim

With these tables to interpret ∧ , ∨ , ¬ → we can calculate the truth-value of any compound claim. For example,

If Dick goes to the movies and Zoe visits her mother,
then no one will walk Spot tonight.

We can formalize this as:

(Dick goes to the movies ∧ Zoe visits her mother) → no one will walk Spot tonight.

I had to use parentheses to mark off the antecedent. They do the work that commas (should) do in ordinary English.

When is this claim true? Let's look at the form of it:

$(A \wedge B) \rightarrow C$

We don't know which of A, B, and C are true and which are false. We have to look at all possibilities to decide when the compound claim is true. We can construct a table:

A	B	C	$A \wedge B$	$(A \wedge B) \rightarrow C$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

We first list all possible values for A, B, and C. Then we calculate the value of $A \wedge B$. With the truth-value of $A \wedge B$ we can use the truth-value of C (to its left in the table) to calculate the truth-value of $(A \wedge B) \rightarrow C$.

We can see now that the original claim can be false only if both "Dick goes to the movies" is true, and "Zoe visits her mother" is true, and "No one will walk Spot tonight" is false. For example, if Dick doesn't go to the movies (A is F) and Zoe doesn't visit her mother (B is F), then the whole claim is true—the antecedent of $(A \wedge B) \rightarrow C$ is false, so the claim is vacuously true.

Perhaps you could have figured out when this claim is true without using a table. But it's equally routine to analyze a complex claim with the complicated form $(\neg(A \wedge B) \vee C) \rightarrow (\neg B \vee (C \wedge \neg A))$.

Some compound claims are true for every way their parts could be true or false. For example:

Ralph is a dog or Ralph isn't a dog.

We formalize this as:

$A \vee \neg A$

Here is the table to evaluate it:

A	$\neg A$	$A \vee \neg A$
T	F	T
F	T	T

It doesn't matter whether A is true or false. Any claim with the form $A \vee \neg A$ is true.

Tautology A compound claim is a *tautology* means that it is true no matter what the truth-values are of the smallest parts of it.

The form $(A \vee B) \rightarrow (B \vee A)$ is a tautology, too, which reflects that the order of the parts of an "or" claim doesn't matter.

A	B	$A \vee B$	$B \vee A$	$(A \vee B) \rightarrow (B \vee A)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

> A claim is a tautology iff in the table for its form *the last column of the table has only* T.

Using tables we can also verify the equivalences of informal claims we noted in the textbook. Recall that two claims are equivalent if they are both true or both false in every possible circumstance. Here, let's use $\equiv$ as shorthand for "is equivalent to."

$A \rightarrow B \quad \equiv \quad \neg B \rightarrow \neg A$ *contrapositive*

$\neg(A \rightarrow B) \quad \equiv \quad A \wedge \neg B$ *contradictory of a conditional*

$\neg A \vee B \quad \equiv \quad A \rightarrow B$ *conditional form of an "or" claim*

For example, no matter what truth-values A and B have, $A \rightarrow B$ is going to have the same truth-value as $\neg B \rightarrow \neg A$.

A	B	$A \rightarrow B$	$\neg B$	$\neg A$	$\neg B \rightarrow \neg A$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Exercises for Sections A and B

1. What are the four English words or phrases we will analyze in studying compound claims?
2. What is the first big assumption about claims we made when we decided to use the symbols $\wedge$, $\vee$, $\neg$, $\rightarrow$?
3. What is a tautology?
4. Describe the method for checking whether a claim is a tautology.
5. Describe the method for checking whether two forms of claims are equivalent.

Here's an example of a method Tom devised to check whether a claim is a tautology.
It's a little long-winded, but it made it clear to him.

Decide whether $(A \wedge B) \rightarrow \neg(A \vee B)$ is a tautology.

A	B		A	∧	B		¬	(A	∨	B)		(A ∧ B)	→	¬(A ∨ B)
T	T		T	T	T		F	T	T	T		T	F	F
T	F		T	F	F		F	T	T	F		F	T	F
F	T		F	F	T		F	F	T	T		F	T	F
F	F		F	F	F		T	F	F	F		F	T	T
1	2		3	4	5		6	7	8	9		10	11	12

Columns 1 and 2 are all the possible combinations of truth-values of the claims.
Columns 3 and 5 are just 1 and 2 repeated to see how to get column 4 (the table for $A \wedge B$).
Columns 7 and 9 are just 1 and 2 repeated so as to see how to get column 8
(the table for $A \vee B$).
Then column 6 is the table for ¬ applied to column 8, which gives the table for $\neg(A \vee B)$.
Column 10 is just column 4 repeated. And column 12 is just column 6 again.
That lets us see how to get column 11 using the table for →.
Column 11 gives the truth-values for $(A \wedge B) \rightarrow \neg(A \vee B)$. Since there's an F in that column, this isn't the form of a tautology.

Use truth-tables to show that the following are tautologies:

6. $\neg\neg A \rightarrow A$
7. $\neg(A \wedge \neg A)$
8. $((A \rightarrow B) \wedge (\neg A \rightarrow B)) \rightarrow B$
9. $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

Decide whether the following are tautologies using truth-tables. Then explain your answer in your own words.

10. $A \rightarrow (A \vee B)$
11. $((A \vee B) \wedge \neg B) \rightarrow A$
12. $(A \vee B) \rightarrow (A \wedge B)$
13. $((A \rightarrow B) \wedge \neg B) \rightarrow \neg A$
14. $(\neg(A \wedge B) \wedge \neg A) \rightarrow B$
15. $((A \rightarrow B) \wedge (\neg A \rightarrow C)) \rightarrow (B \vee C)$

Using truth-tables, show that the following are equivalent.

16. $\neg(A \rightarrow B)$ is equivalent to $A \wedge \neg B$
17. $A \rightarrow B$ is equivalent to $\neg A \vee B$
18. $\neg(A \wedge B)$ is equivalent to $\neg A \vee \neg B$
19. $\neg(A \vee B)$ is equivalent to $\neg A \wedge \neg B$

C. Representing Claims

To use truth-tables we have to be able to represent ordinary claims and arguments.

Examples Can the following be represented in a form that uses ¬, →, ∧, ∨ ?

Example 1 Spot is a dog or Puff is a cat and Zoe is not a student.

Analysis What's the form of this? (A ∨ B) ∧ ¬C ? or A ∨ (B ∧ ¬C) ? Without a context, we have to guess. We analyze the argument on one reading, then on the other, and see which is better. Our formal analyses help us see ambiguities.

Example 2 Puff is a cat or someone got swindled at the pet store.

Analysis This one's easy: Puff is a cat ∨ someone got swindled at the pet store.

Example 3 London is in England or Paris is in France.

Analysis We could represent this using *exclusive* "or":

(London is in England ∨ Paris is in France) ∧
¬ (London is in England ∧ Paris is in France)

In Exercise 1 I ask you to show:

(A ∨ B) ∧ ¬ (A ∧ B) is true iff exactly one of A is true or B is true.

Example 4 Harry is a football player if he plays any sport at all.

Analysis We're used to rewriting conditionals. This one is:

If Harry plays any sport at all, then Harry is a football player.

Harry plays any sport at all → Harry is a football player.

Example 5 Zoe loves Dick although he's not a football player.

Analysis This is a compound claim, with parts "Zoe loves Dick" and "Dick is not a football player." But "although" isn't one of the words we're formalizing.

When is this compound claim true? If we stick to the classical abstraction, then "although" doesn't do anything more than "and." It shows that the second part is perhaps surprising, but that isn't what we're paying attention to. We can formalize the claim as:

Zoe loves Dick ∧ Dick is not a football player

If all we're interested in is whether the argument in which this appears is valid, this representation will do.

There are a lot of words or phrases that can sometimes be represented with ∧ :

and
but
even if
although
even though
despite that

Sometimes, though, these serve as indicator words, suggesting the roles of the claims in the overall structure of the argument. "Even though" can indicate that the claim is going to be used as part of a counterargument. We can represent these words or phrases with $\wedge$, or we can just represent the parts of the sentence as separate claims. That's what we did in the textbook, and we can do that because the table for $\wedge$ says that the compound will be true iff both parts are true.

Example 6 Spot thinks that Dick is his master because Zoe doesn't take him for walks.

Analysis Can we represent "because" using $\wedge$, $\vee$, $\neg$, $\rightarrow$? Consider the following two claims:

> Spot is a dog because Las Vegas is in the desert.
>
> Spot is a dog because Las Vegas is not in the desert.

Both of these are false. Spot is a dog, and that's true whether Las Vegas is or is not in the desert. The truth-value of "Las Vegas is in the desert" is irrelevant to the truth-value of the whole compound. Yet all we've got to work with in representing "because" are compounds that depend on whether the parts are true or false. We can't represent this example as a compound claim.

Example 7 Zoe took off her clothes and went to bed.

Analysis We shouldn't represent this compound as:

> Zoe took off her clothes $\wedge$ Zoe went to bed.

That has the same truth-value as:

> Zoe went to bed $\wedge$ Zoe took off her clothes.

Example 7 is true most nights, but "Zoe went to bed and took off her clothes" is false. In this example "and" has the meaning "and then next," so that *when* the claims become true is important. But if we use these symbols we can only consider whether the claims are true, not when they become true. So we can't represent this claim.

Example 8 (On the playground): Hit me and I'll hit you.

Analysis We don't represent this as: You hit me $\wedge$ I hit you. We understand it to mean "If you hit me, then I'll hit you," and we represent it as:

> You hit me $\rightarrow$ I hit you

To represent a use of "and", "or", "not", or "if . . . then . . ." as $\wedge$, $\vee$, $\neg$, $\rightarrow$, we have to ask what the words mean in the way they're used. Does the use accord with the classical abstraction?

Exercises for Section C

1. Make up the table for $(A \vee B) \wedge \neg(A \wedge B)$ and show that it is true when exactly one of A, B is true.

For each of the following, either represent it using $\wedge$, $\vee$, $\neg$, $\rightarrow$, or explain why it can't be represented.

2. If critical thinking is hard, then mathematics is impossible.
3. If you don't apologize, I'll never talk to you again.
4. Dick prefers steak, while Zoe prefers spaghetti.
5. Dick was shaving while Zoe was preparing dinner.
6. Either Dick loves Zoe best, or he loves Spot best.
7. Even if you do whine all the time, I love you.
8. Spot is a good dog even though he scared the living bejabbers out of your cat.
9. Spot is a good dog because he scared the living bejabbers out of your cat.
10. We're going to go to the movies or go out for dinner tonight.
11. Since 2 + 2 is 4, and 4 times 2 is 8, I should be ahead $8, not $7, in blackjack.
12. If Dick has a class and Zoe is working, there's no point in calling their home to ask them over for dinner.
13. If it's really true that if Dick takes Spot for a walk he'll do the dishes, then Dick won't take Spot for a walk.
14. If Dick goes to the basketball game, then he either got a free ticket or he borrowed money from somebody.
15. Either we'll go to the movies or visit your mom if I get home from work by 6.
16. Whenever Spot barks like that, there's a skunk or raccoon in the yard.
17. I'm not going to visit your mother and I'm not going to do the dishes, regardless of whether you get mad at me or try to cajole me.
18. Every student in Dr. E's class is over 18 or is taking the course while in high school.
19. No matter whether the movie gets out early or late, we're going to go out for pizza.
20. Suggest ways to represent:
    a. A only if B
    b. A unless B
    c. When A, B
    d. A if and only if B
    e. B just in case A
    f. Neither A nor B

D. Checking for Validity

An argument is valid if for every possible way the premises could be true, the conclusion is true, too. For example, suppose we have an argument of the form:

$A \to B$
$\neg A \to B$
So B.

For an argument of this form to be valid, it has to be impossible that $A \to B$ and $\neg A \to B$ are both true, and B is false. We need to look at all ways that $A \to B$ and $\neg A \to B$ could be true:

A	B	$A \to B$	$\neg A$	$\neg A \to B$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

We list all the values of A and B. Then we calculate the truth-values of $A \to B$ and $\neg A \to B$. In the first row both of those are true, and so is the conclusion, B. Ditto for the third row. In the second row $A \to B$ is false, and we don't care about that. In the last row $\neg A \to B$ is false, and we can ignore that. So whenever both $A \to B$ and $\neg A \to B$ are true, so is B. Any argument of this form is valid.

Valid argument form An argument form is valid if every argument of that form is valid.

We can show that an argument form is valid by making a table that includes all the premises and the conclusion. If in every row in which all the premises are true, the conclusion is true, too, then the form is valid.

Let's look at the indirect way of reasoning with conditionals:

$A \to B$
$\neg B$
So $\neg A$.

Again, we have to look at every way the premises could be true.

A	B	$A \to B$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Only in the last row are both premises $A \to B$ and $\neg B$ true. There we find that $\neg A$ is true, too. So every argument of this form is valid.

The third row of this table also shows that, in contrast, denying the antecedent is invalid:

$A \to B$
$\neg A$
So $\neg B$.

Both $A \to B$ and $\neg A$ are true, but $\neg B$ is false. It is possible to have the premises true and the conclusion false.

Reasoning in a chain provides a more complicated example:

$A \to B$
$B \to C$
So $A \to C$.

We have the table:

A	B	C	$A \rightarrow B$	$B \rightarrow C$	$A \rightarrow C$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

I've circled the rows in which both premises are true. In each of them the conclusion is also true. So every argument of this form is valid.

This last table also shows that the following form isn't valid:

$A \rightarrow B$
$A \rightarrow C$
So $B \rightarrow C$.

The third row from the bottom has both $A \rightarrow B$ and $A \rightarrow C$ true, with $B \rightarrow C$ false.

So far this has been just a game, playing with symbols. It's only when we can apply these tables to real arguments that we're doing critical thinking. Consider:

If Tom knows some logic, Tom is either very bright or he studies hard.
Tom is bright. Tom studies hard. So Tom knows some logic.

First we represent these claims. Only the first is a compound claim:

Tom knows some logic $\rightarrow$ (Tom is very bright $\vee$ Tom studies hard)

So the argument has the form:

$A \rightarrow (B \vee C)$
B
C
So A.

A	B	C	$B \vee C$	$A \rightarrow (B \vee C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

I've circled a row in which all of $A \rightarrow (B \vee C)$, B, and C are true, yet the conclusion A is false. So the argument is not valid.

That alone does not make it a bad argument. We have to see if it could be strong. But this argument isn't even strong: though Tom is very bright and studies hard, and the first premise is true too, it's not unlikely that Tom could be majoring in art history and knows no logic at all.

You might not have needed a table to figure out this last one. But you will for some of the exercises. Have fun.

Exercises for Section D

1. What does it mean to say an argument form is valid?

2. If an argument has a form that is not valid, is it necessarily a bad argument?

Use truth-tables to decide whether the following argument forms are valid.

3. $A \rightarrow B$
   B
   So A.

4. $A \rightarrow B$
   $A \rightarrow \neg B$
   So $\neg A$.

5. A
   $\neg A$
   So B.

6. $A \vee B$
   So $A \wedge B$.

7. $A \vee B$
   $\neg A$
   So B.

8. $B \vee D$
   $B \rightarrow C$
   $D \rightarrow E$
   So $C \vee E$.

9. $A \rightarrow \neg B$
   $B \wedge \neg C$
   So $A \rightarrow C$.

10. $A \rightarrow \neg\neg B$
    $\neg C \vee A$
    C.
    So B.

Represent the arguments in the following exercises and decide whether they are valid.
Use truth-tables or not as you wish.

11. If Spot is a cat, then Spot meows. Spot is not a cat. So Spot doesn't meow.

12. Either the moon is made of green cheese or 2 + 2 = 4. But the moon is not made of green cheese. So 2 + 2 = 4.
13. Either the moon is made of green cheese or 2 + 2 = 5. But the moon is not made of green cheese. So 2 + 2 = 5.
14. The students are happy if and only if no test is given. If the students are happy, the professor feels good. But if the professor feels good, he won't feel like lecturing, and if he doesn't feel like lecturing, he'll give a test. So the students aren't happy.
15. If Dick and Zoe visit his family at Christmas, then they will fly. If Dick and Zoe visit Zoe's mother at Christmas, then they will fly. But Dick and Zoe have to visit his family or her mother. So Dick and Zoe will travel by plane.
16. Tom is not from New York or Virginia. But Tom is from the East Coast. If Tom is from Syracuse, he is from New York or Virginia. So Tom is not from Syracuse.
17. The government is going to spend less on health and welfare. If the government is going to spend less on health and welfare, then either the government is going to cut the Medicare budget or the government is going to slash spending on housing. If the government is going to cut the Medicare budget, the elderly will protest. If the government is going to slash spending on housing, then advocates of the poor will protest. So the elderly will protest or advocates of the poor will protest.

Summary By concentrating on just whether claims are true and the structure of arguments that involve compound claims, we can devise a method for checking the validity of arguments. We introduced symbols for the words "and", "or", "not", and "if . . . then . . ." and made precise their meaning through truth-tables. We learned how to use the symbols and tables in representing claims. Then we saw how to use truth-tables to check whether the structure of an argument relative to the compound claims in it is enough to guarantee that the argument is valid.

Key Words	classical abstraction	conjunction
	truth-table	negation
	$\wedge$	disjunction
	$\neg$	conditional
	$\vee$	tautology
	$\rightarrow$	valid argument form

Further Study For a fuller study of the formal logic of reasoning with compound claims, see *An Introduction to Formal Logic*, also published by ARF.

Answers to Selected Exercises

Sections A and B

1. “and”, “or”, “not”, “if . . . then . . .”.
2. The only aspects of claims that we will pay attention to are whether the claim is true or false and how it is compounded out of other claims.
3. A compound claim that is true regardless of the truth-values of its parts.
4. Represent the claim using $\wedge$, $\vee$, $\neg$, $\rightarrow$. Replace the claims with letters. Make a truth-table with the last column the formal claim. If all the entries in that column are T, then it’s a tautology. If even one is F, it’s not a tautology.
5. Form the table for each. They are equivalent if for every row they are both true or both false.

6.

A	$\neg\neg$A	$\neg\neg$A $\rightarrow$ A
T	T	T
F	F	T

8.

A	B	A $\rightarrow$ B	$\neg$A	$\neg$A $\rightarrow$ B	(A $\rightarrow$ B) $\wedge$ ($\neg$A $\rightarrow$ B)	((A $\rightarrow$ B) $\wedge$ ($\neg$A $\rightarrow$ B)) $\rightarrow$ B
T	T	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

11. Tautology.

A	B	A $\vee$ B	$\neg$B	(A $\vee$ B) $\wedge$ $\neg$B	((A $\vee$ B) $\wedge$ $\neg$B) $\rightarrow$ A
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

12. Not a tautology.

A	B	A $\vee$ B	A $\wedge$ B	(A $\vee$ B) $\rightarrow$ (A $\wedge$ B)
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

14. Not a tautology.

A	B	A $\wedge$ B	$\neg$(A $\wedge$ B)	$\neg$A	$\neg$(A $\wedge$ B) $\wedge$ $\neg$A	($\neg$(A $\wedge$ B) $\wedge$ $\neg$A) $\rightarrow$ B
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	F	T	T	T	T
F	F	F	T	T	T	F

15. Not a tautology.

A	B	C	$A \to B$	$\neg A$	$\neg A \to C$	$(A \to B) \vee (\neg A \to C)$	$B \vee C$	$((A \to B) \vee (\neg A \to C)) \to (B \vee C)$
T	T	T	T	F	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	F	T	T	T	T
T	F	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	T	F	F

16.

A	B	$A \to B$	$\neg(A \to B)$	$\neg B$	$A \wedge \neg B$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

18.

A	B	$A \wedge B$	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$\neg A \vee \neg B$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Section C

1.

A	B	$A \vee B$	$A \wedge B$	$\neg(A \wedge B)$	$(A \vee B) \wedge \neg(A \wedge B)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

4. Dick prefers steak $\wedge$ Zoe prefers spaghetti . Here "while" doesn't mean "at the same time."
5. Here "while" does "at the same time," yet when a claim is true can't matter for the use or our symbols. So the claim can't be represented.
7. Not compound, just two claims. Or: You whine all the time $\wedge$ I love you.
10. We're going to the movies tonight $\vee$ we're going out for dinner tonight.
    However, if you think the "or" is exclusive then follow Example 3.
13. (Dick takes Spot for a walk $\to$ Dick will do the dishes) $\to$ $\neg$(Dick will take Spot for a walk)
16. Spot barks $\to$ (there's a skunk in the yard $\vee$ there's a raccoon in the yard)
17. How you formalize this will depend on how you understand "regardless." Here's one interpretation:

[(You get mad at me) → ¬(I will visit your mother)] ∧ ¬(You get mad at me) → ¬(I will visit your mother) ∧ (You cajole me) → ¬(I will visit your mother) ∧ ¬(You cajole me) → ¬(I will visit your mother)

18. Can't represent it. It's *not*: Every student in Dr. E's class is over 18 ∨ every student in Dr. E's class is taking the course while in high school. That could be false and the original true. [Compare: Every student is male or female.]

20. a. A → B

 b. ¬B → A (This is the same as ¬A → B.)

 c. A → B if "when" doesn't mean "at that time."

 d. (A → B) ∧ (B → A)

 e. B → A

 f. ¬A ∧ ¬B

Section D

1. Every argument that has that form is valid.
2. No. It might have another valid argument form. Or it might be a strong argument. For example: "All cats meow. Puff is a cat. So Puff meows." Truth-tables won't show that this is valid.

4. Valid.

A	B	A → B	¬B	A → ¬B	¬A
T	T	T	F	F	F
T	F	F	T	T	F
F	T	(T	F	T	T)
F	F	(T	T	T	T)

6. Invalid. Either circled row shows that.

A	B	A ∨ B	A ∧ B
T	T	T	T
T	F	(T	F)
F	T	(T	F)
F	F	F	F

8. Valid. Argue that if C ∨ E were false, then both C and E would be false. Since B → C and D → E are true, both B and D would have to be false. But B ∨ D is true. So one of B or D is true. That's a contradiction. So one of C or E is true.

9. Valid.

A	B	C	¬B	A → ¬B	¬C	B ∧ ¬C	A → C
T	T	T	F	F	F	F	T
T	T	F	F	F	T	T	F
T	F	T	T	T	F	F	T
T	F	F	T	T	T	F	F
F	T	T	F	T	F	F	T
F	T	F	F	(T	T	T	T)
F	F	T	T	T	F	F	T
F	F	F	T	T	T	F	T

10. Valid.

A	B	C	$\neg B$	$\neg\neg B$	$A \rightarrow \neg\neg B$	$\neg C$	$\neg C \vee A$
T	T	T	F	T	T	F	T
T	T	F	F	T	T	T	T
T	F	T	T	F	F	F	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	T	F	F	T	T	T	T
F	F	T	T	F	T	F	F
F	F	F	T	F	T	T	T

11. Spot is a cat $\rightarrow$ Spot meows, $\neg$ (Spot is a cat)
    So $\neg$(Spot meows)
    $A \rightarrow B,\ \neg A$
    $\neg B$
    Not valid. Denying the antecedent.

12. The moon is made of green cheese $\vee$ 2 + 2 = 4
    $\neg$(the moon is made of green cheese)
    So 2 + 2 = 4
    $A \vee B,\ \neg A$
    B
    Valid. Excluding possibilities.

13. Valid, same as Exercise 12. Sure, the conclusion is false. It's valid, not good.

14. (The students are happy $\rightarrow$ no test is given) $\wedge$ (no test is given $\rightarrow$ the students are happy)
    The students are happy $\rightarrow$ the professor feels good
    The professor feels good $\rightarrow$ $\neg$(the professor will feel like lecturing)
    $\neg$(the professor feels like lecturing) $\rightarrow$ the professor will give a test
    So $\neg$(the students will be happy)
    [Identify "the professor will give a test" with "a test is given."]
    $(A \rightarrow \neg B) \wedge (\neg B \rightarrow A),\ A \rightarrow C,\ C \rightarrow \neg D,\ \neg D \rightarrow B$
    $\neg A$
    First note that the last three premises yield, via reasoning in a chain, $A \rightarrow B$.
    So we've reduced it to: $(A \rightarrow \neg B) \wedge (\neg B \rightarrow A),\ \ A \rightarrow B$
    $\neg A$
    Thus we have both $A \rightarrow B$ and $A \rightarrow \neg B$. And so from Exercise 4 we have $\neg A$.
    So it's valid.

15. Dick and Zoe visit his family at Christmas $\rightarrow$ they will fly
    Dick and Zoe visit Zoe's mother at Christmas $\rightarrow$ they will fly
    Dick and Zoe visit his family at Christmas $\vee$ Dick and Zoe visit Zoe's mother at Christmas
    So: they will fly
    [Identify "They will fly" with "Dick and Zoe will travel by plane."]
    $A \rightarrow B,\ C \rightarrow B,\ A \vee C$
    B

A	B	C	A → B	C → B	A ∨ C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Valid.

16. ¬(Tom is from NY ∨ Tom is from Virginia)
Tom is from Syracuse → (Tom is from NY ∨ Tom is from Virginia)
So: ¬ (Tom is from Syracuse)
["Tom is from the East Coast" isn't needed.]
¬(B ∨ C) , D → (B ∨ C)
¬D
Valid.
If it were possible to have ¬D false, and so D true, with these premises true, then by the direct way of reasoning with conditionals, B ∨ C would be true. But the first premise gives us that ¬(B ∨ C) is true. A contradiction. So D is false. So ¬D has to be true.

17. The government is going to spend less on health and welfare.
The government is going to spend less on health and welfare →
(the government is going to cut the Medicare budget ∨ the government is going to slash spending on the elderly)
The government is going to cut the Medicare budget → the elderly will protest
The government is going to slash spending on the elderly → advocates of the poor will protest
So: The elderly will protest ∨ advocates of the poor will protest
A, A → (B ∨ C) , B → D, C → E
D ∨ E
Valid. From the first two premises we get B ∨ C. Then if we want we can do a table. Or we can argue as follows. Suppose D ∨ E were false. Then both D and E are false. So by the indirect way of reasoning with conditionals, both B and C would have to be false. So B ∨ C would have to be false. But we already have that B ∨ C is true. So we can't have D ∨ E false.